

ZU–TH 41/99

On the reordering of Baryon Chiral Perturbation Theory*

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Abstract

Reordering of the chiral perturbation series, proposed recently by Becher and Leutwyler in the framework of $SU(2)$ baryonic ChPT, is applied to the $SU(3)$ case. This results in improved convergence of the chiral expansion of static properties of the lowest lying baryon octet, which in most cases is quite impressive. Finite renormalization of coupling constants and the role it plays in the interpretation of effective field theories is discussed. Some future tests of the viability of the scheme are proposed too.

* Work supported in part by VEGA grant No.1/4301/97, by Schweizerischer Nationalfonds and by the EEC-TMR Program, Contract No. CT98-0169.

1. The phenomenology of baryons at low energies is reasonably well described by the chiral symmetry $SU(3)_L \times SU(3)_R$, broken spontaneously to $SU(3)_V$, as well as its explicit breaking by non-vanishing quark masses. The most celebrated successes of chiral $SU(3)$ symmetry go back to the sixties. These are now understood as tree level results of chiral perturbation theory (ChPT) — the low-energy effective theory of QCD. The main virtue of ChPT is that it enables calculation of loop corrections to these tree results in a systematic way. However, the loop-corrections turn out to be typically too large, thereby corrupting the overall good agreement between theory and experiment at tree level. The agreement can be restored by adjusting large counterterms at higher orders, but it is commonly believed that this happens only at the expense of a slow convergence of the perturbation series.

To illustrate the problem, let us quote the results for mass [1] and magnetic moment of the Λ -hyperon [2]

$$\begin{aligned} m_\Lambda &= 767 (1 + 0.69 - 0.77 + 0.54) \text{ MeV} \\ \mu_\Lambda &= -1.31 (0 + 1 - 0.82 + 0.29) \mu_N \end{aligned} \tag{1}$$

where $\mu_N = \frac{e\hbar}{2m_p}$ is the nucleon magneton. Here, as well as in formulae to follow, the numbers in brackets correspond to contributions from different chiral orders (from the 1st up to the 4th). The pattern is characteristic for other quantities, like e.g. weak decay constants, as well as for all members of the baryon octet. The problem of slow convergence of the static baryon properties was recently studied in [3], where a remedy called long distance regularization (a variant of cut-off regularization) was proposed.

It is, however, questionable whether the direct comparison of various contributions in the chiral expansion of a given observable is the best test of the rate of convergence. ChPT is, after all, the tool for solving Ward identities imposed by chiral symmetry, i.e. it *relates* physical observables which would be independent in the absence of the symmetry. It is therefore more natural to form an opinion about the rate of convergence from changes in such relations, at each order of the chiral expansion, rather than from expansions like (1). This point of view was emphasized recently in [4], where a reordering of the perturbation series was proposed too. This reordering allows to judge the rate of convergence from expansions of individual quantities like (1), but is in spirit much closer to the comparison of relations between different physical observables.

The essence of the reordering is very simple and well known: results are expressed not in terms of bare coupling constants, but rather in terms of

the physical couplings. This amounts to replacing the bare constants by the physical ones and shifting the difference to higher orders. For rapidly converging series, this is just useful cosmetics. For slowly convergent series, the reordering may be quite significant. As an example let us quote the expansion of the πN σ -term as given in [4]. The shift due to the loop corrections amounts numerically to -31 MeV in the expansion analogous to (1), but only to 2 MeV in the reordered series.

Why is the reordered expansion preferable? Consider the process of pinning down values of low energy constants (LECs) from experimental data of quantities forming a set A and then using these values in results for quantities forming a set B . This is nothing else than relating the quantities from the sets A and B . In other words, the relations between various quantities are embodied in the values of the LECs. Tree level relations are characterized by values pinned down at the tree level, etc. Comparison of relations between various quantities at 1-loop and tree level corresponds to comparison of values of LECs pinned down at the 1-loop and tree level. This is what is done in the reordered perturbation series. On the other hand, expansions like (1) are comparing values of different orders, as given by the fit using the complete result up to the highest order calculated. In such an expansion, the values of counterterms at lower orders do not characterize relations between physical observables.

In [4] the reordered perturbation series was discussed in the framework of $SU(2)$ baryonic ChPT. However, it is clear that reordering could be even more relevant in the $SU(3)$ case. The problem with $SU(3)$ baryonic ChPT, as stressed in [3], is that the simplicity evident in the baryon physics at tree level is lost at the one-loop level. The reordered perturbation series has an immediate benefit: tree level results are not changed after loop corrections are included. The aim of this note is to investigate to what extent can the reordered perturbation series help to cure the problem of slow convergence in $SU(3)$ baryonic ChPT.

2. Reordering of the perturbation series discussed in [4] was reordering between different orders in the *loop* expansion. Expansion (1) and other examples collected in [3] are more detailed — they treat separately every chiral order. To be able to investigate the influence of the reordering of perturbation series on these more detailed expansions, we slightly generalize the approach of [4] and perform reordering between different orders of the *chiral* expansion.

Formally this is achieved by writing any 1st order LEC c in the form

$$c = c^{(0)} + \delta c^{(1)} + \delta c^{(2)} + \delta c^{(3)} + \dots \quad (2)$$

any 2nd order LEC d in the form

$$d = d^{(0)} + \delta d^{(1)} + \delta d^{(2)} + \dots \quad (3)$$

etc. Here the superscript (n) refers to the chiral order (rather than to the number of loops, as in [4]). The standard chiral expansion of the meson-baryon Lagrangian

$$\mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \dots \quad (4)$$

is then rewritten in the form

$$\mathcal{L}_{MB} = \underbrace{\mathcal{L}_{MB}^{(1)} |_{c \rightarrow c^{(0)}}}_{\text{1st order}} + \underbrace{\mathcal{L}_{MB}^{(2)} |_{d \rightarrow d^{(0)}} + \mathcal{L}_{MB}^{(1)} |_{c \rightarrow \delta c^{(1)}}}_{\text{2nd order}} + \dots \quad (5)$$

It is now straightforward to reorder the chiral expansion of any quantity. Our starting point is the Lagrangian

$$\begin{aligned} L_{MB}^{(1)} &= \text{Tr } \bar{B} \gamma^\mu [i \nabla_\mu, B] - m \text{Tr } \bar{B} B \\ &+ \frac{1}{2} d_A \text{Tr } \bar{B} \gamma^\mu \gamma^5 \{u_\mu, B\} + \frac{1}{2} f_A \text{Tr } \bar{B} \gamma^\mu \gamma^5 [u_\mu, B] \end{aligned} \quad (6)$$

$$\begin{aligned} L_{MB}^{(2)} &= b_0 \text{Tr } \chi_+ \text{Tr } \bar{B} B + d_m \text{Tr } \bar{B} \{\chi_+, B\} + f_m \text{Tr } \bar{B} [\chi_+, B] \\ &+ d_V \text{Tr } \bar{B} \sigma_{\mu\nu} \{F_+^{\mu\nu}, B\} + f_V \text{Tr } \bar{B} \sigma_{\mu\nu} [F_+^{\mu\nu}, B] + \dots \end{aligned} \quad (7)$$

where $\chi_+ = 4B_0 \text{diag}(m_u, m_d, m_s) + \dots = 2 \text{diag}(M_\pi^2, M_\pi^2, 2M_K^2 - M_\pi^2) + \dots$ and ellipses always stand for terms not needed in what follows. We refrain from giving the explicit form of the relevant terms in $L_{MB}^{(3)}$ and $L_{MB}^{(4)}$, they can be found in the quoted papers [1] [2].

To illustrate how the reordering works, let us consider the mass of the Λ -hyperon. At tree level the result is straightforward

$$m_\Lambda^{\text{tree}} = m - 2(2M_K^2 + M_\pi^2) b_0 - \frac{4}{3}(4M_K^2 - M_\pi^2) d_m. \quad (8)$$

We have replaced the bare meson masses by the physical ones, does not affect the result up to the 4th order. Moreover, this procedure is in spirit of the considered reordering. At one-loop level one has

$$m_\Lambda = m_\Lambda^{\text{tree}} + L_3(m, d_A, f_A) + L_4(\dots), \quad (9)$$

where the explicit form of the loop corrections L_3 and L_4 can be found in [1]. We just recall that L_3 is a finite loop correction (no counterterms of the 3rd order), while L_4 contains loop as well as counterterm contributions containing 4th order LECs. The latter as well as some lower order LECs are represented by the ellipses in (9). Then the reordered series assumes the form

$$\begin{aligned} m_\Lambda &= m^{(0)} \\ &+ \delta m^{(1)} - 2(2M_K^2 + M_\pi^2) b_0^{(0)} - \frac{4}{3}(4M_K^2 - M_\pi^2) d_m^{(0)} \\ &+ \delta m^{(2)} - 2(2M_K^2 + M_\pi^2) \delta b_0^{(1)} - \frac{4}{3}(4M_K^2 - M_\pi^2) \delta d_m^{(1)} + L_3(m^{(0)}, d_A^{(0)}, f_A^{(0)}) \\ &+ \dots \end{aligned} \quad (10)$$

where the n-th line represents the n-th order of the reordered series.

Analogous expansions hold for other members of the lowest lying baryon octet. The numerical value of $m^{(0)}$ is obtained by fitting the experimental values of baryon masses to the 1st order results. This yields $m^{(0)} = 1150$ MeV. Numerical values of $\delta m^{(1)}$, $b_0^{(0)}$ and $d_m^{(0)}$ are then obtained by the fit with second order results, using for $m^{(0)}$ the already fixed value 1150 MeV. In a similar way, the values of $\delta m^{(2)}$, $\delta b_0^{(1)}$ and $\delta d_m^{(1)}$ are determined by the 3rd order fit. Here one uses the values for the LECs d_A and f_A determined via semileptonic weak decays of baryons. The data are well reproduced already at the tree level, with $d_A^{(0)} = 0.75$ and $f_A^{(0)} = 0.5$. At the 4th order a number of LECs enter the result. In [1] these were estimated by the resonance saturation principle. Using these estimates for the 4th order results one obtains the following reordered chiral expansion of baryon masses

$$\begin{aligned} m_N &= (1150 - 209 + 0 + 3) \text{ MeV} \\ m_\Lambda &= (1150 - 39 + 1 + 8) \text{ MeV} \\ m_\Sigma &= (1150 + 39 + 0 + 0) \text{ MeV} \\ m_\Xi &= (1150 + 170 + 0 + 0) \text{ MeV}. \end{aligned} \quad (11)$$

Comparison with (1) as well as with expansions of m_N , m_Σ and m_Ξ as given in [1] shows a tremendous improvement in the rate of convergence.

At first sight this improvement may look too good. For a long time $SU(3)$ ChPT has had a reputation of being a rather slowly convergent theory, so the result (11), and especially the zeros in the third and fourth orders may come as a surprise. However, there is nothing mysterious here. All we have done is to implement a fitting procedure such that higher orders cannot spoil a prospective agreement achieved at lower orders. And since there is very good agreement already at the 2nd order, higher corrections must be tiny. Of course, the large corrections have not disappeared completely but are now hidden, e.g. in the expansion of m . Note, however, that m is not a physical observable.

A natural objection is that we have made convergence more rapid by hand, i.e. by reshuffling a (large) part of the originally second order contribution to higher orders, where it has cancelled the large loop contributions. The objection is valid, but this is possible only for a small number of quantities, equal to the number of LECs to the order considered. In our case we have four LECs, but two of them, namely m and b_0 enter only in a particular combination, so the number is effectively reduced to three. And since the convergence was improved for four masses, this is already a nontrivial result. Moreover, one can now study other quantities depending on the same set of LECs. The question then is, if the convergence of these quantities in the reordered version of the perturbation series is improved too.

3. In case of the baryon magnetic moments the reordering is a consequence of the expansion (3) of LECs d_V and f_V . Performing the fit at various orders of the chiral expansion (relevant formulae are found in [2]) one obtains the following reordered chiral expansion of baryon magnetic moments

$$\begin{aligned}
\mu_p &= (2.56 + 0.39 - 0.15) \mu_N \\
\mu_n &= (-1.60 - 0.91 + 0.60) \mu_N \\
\mu_\Lambda &= (-0.80 + 0.36 - 0.18) \mu_N \\
\mu_{\Sigma^+} &= (2.56 - 0.33 + 0.22) \mu_N \\
\mu_{\Sigma^-} &= (-0.97 - 0.39 + 0.19) \mu_N \\
\mu_{\Sigma^0} &= (0.80 - 0.36 + 0.22) \mu_N \\
\mu_{\Xi^0} &= (-1.60 + 0.79 - 0.45) \mu_N \\
\mu_{\Xi^-} &= (-0.97 + 0.45 - 0.15) \mu_N.
\end{aligned} \tag{12}$$

Here the first number in brackets represents the sum of the first two chiral orders, and is followed by the third and the fourth order contribution. Comparison with (1) as well as with expansions of other magnetic moments as

given in Eq. (29) of [2] shows again a (moderate) improvement in the rate of convergence. From the point of view of the loop expansion, the convergence is even better, since the loop contribution (including counterterms from $L_{MB}^{(3)}$ and $L_{MB}^{(4)}$) is given by the sum of the last two numbers in brackets, which are always of comparable size and opposite sign. The loop corrections are typically at the level of 20% of the tree result (4% in the best, 31% in the worst case), which is quite acceptable.

Let us stress that Eq. (12) is not new and practically equivalent to Table 1. in [2] (as it should be, since this Table 1. just presents magnetic moments calculated in various orders). Our point is that to judge the rate of convergence for baryon magnetic moments one should consider results of [2] in the form of their Table 1. (as we have done in this article), rather than in the form of their Eq. (29) (as was done in [3]).

4. The examples discussed so far are the only ones where the complete 1-loop calculation is available. Our reordering between different chiral orders enables us to study also those quantities for which results are presently known only up to the 3rd order. As an illustration we take another example from [3], namely baryon axial couplings. Here the 3rd chiral order 1-loop contributions are once again too large [6, 7]. As an illustration let us quote the expansion of the Λ decay constant

$$g_1 [p\Lambda] (0) = -0.5 - 0.48 \quad (13)$$

A similar pattern is observed in all the other cases considered in [7]. The first term in (13) corresponds to the first two chiral orders. The second term is the 3rd order contribution, however it is not complete. In this case 3rd order LECs of numerous counterterms contributing to axial form factors were neither pinned down from data, nor were they estimated by the resonance saturation principle. Instead, they were (on purpose) ignored together with the analytic part of loop integrals, for details see [7]. Even the incomplete result (13) used to be considered another example of a rather slowly convergent $SU(3)$ chiral expansion.

Reordering of the perturbation series, which amounts here to the expan-

sion (2) of the LECs d_A and f_A , results again in a much improved pattern

$$\begin{aligned}
g_1[pn](0) &= 1.30 - 0.14 \\
g_1[\Lambda\Sigma^-](0) &= 0.65 - 0.01 \\
g_1[p\Lambda](0) &= -0.94 - 0.04 \\
g_1[\Lambda\Xi^-](0) &= 0.29 + 0.02 \\
g_1[n\Sigma^-](0) &= 0.30 + 0.02 \\
g_1[\Sigma^0\Xi^-](0) &= 0.92 + 0.13
\end{aligned} \tag{14}$$

When compared with [7], an impressive improvement is witnessed. Let us recall that the reason of this improvement is simple. If any set of quantities is relatively well described at two different chiral orders, then corrections in the reordered perturbation series are small.

A dramatic change in the rate of convergence of the physical quantities is reflected by a dramatic change in the values of LECs as pinned down at different chiral orders. The values of d_A and f_A obtained at tree level and the (incomplete) 1-loop level are different by a factor of two. It was pointed out in [8] that values of d_A and f_A as obtained from the (incomplete) 1-loop fit reduce the large loop contributions in the case of magnetic moments and are therefore preferable in loop calculations. We emphasize that this philosophy is exactly opposite to the one advocated here.

The problem with the approach [8] is that the values of d_A and f_A are probably to be changed again significantly when going one order higher. In $SU(2)$ baryon ChPT, where the full 1-loop result for g_A is available [9], a large correction of about 25% is present at the 4th chiral order. This would change the whole numerical analysis of [8]. The advantage of the reordered ChPT is that it leaves lower orders untouched after higher orders have been calculated and included.

5. In conclusion, we have applied the reordering of ChPT, proposed by Becher and Leutwyler [4], to the baryon octet static properties. We have shown that the rate of convergence of the chiral expansion is significantly (in some cases tremendously) improved. This does neither prevent large loop corrections nor avoid large cancellations in higher orders. However, in the reordered series the bulk of these large cancellations is due to the natural cancellations caused by the reordering but not to the fine tuning of higher order LECs.

The arguments given in this note are not sufficient to show that reordering of the perturbation series always leads to a rapidly converging series. This question depends on the process considered and has to be studied for further applications in the future. However, we have shown that the examples considered in [3] do not imply there is a problem with the rate of convergence. Further processes which could shed light on the question considered here include non-leptonic hyperon decays, hyperon polarizabilities, strange form factors of the nucleon, Goldberger-Treiman discrepancy as well as kaon photo-photo production. We plan to come back to these topics in a future publication.

Acknowledgements. We are grateful to Gerhard Ecker for careful reading of the manuscript and useful comments.

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